CHAOS 5391 ARTICLE IN PRESS No. of Pages 9
3 February 2007 Disk Used

Available online at www.sciencedirect.com
ScienceDirect

Chaos, Solitons and Fractals xxx (2007) xxx–xxx

CHAOS SOLITONS & FRACTALS

www.elsevier.com/locate/chaos

Regulating complex dynamics in firms and economics systems

Michael Kopel [a,*], Frank Westerhoff [b], Cristian Wieland [c]

[a] Vienna University of Technology, Institute of Management Science, Theresianumgasse 27, A-1040 Vienna, Austria
[b] University of Osnabrueck, Department of Economics, Germany
[c] RAAD Consult, Germany

Accepted 2 January 2007

Communicated by Prof. S. Yousofi

Abstract

The decisions of economic firms are often guided by simple routines. We demonstrate that such a routine may generate suboptimal chaotic dynamics, where the suboptimality is due to the fact that decision outcomes are in regions of low performance. We discuss a simple, but effective, method to regulate such dynamics in order to improve the performance. The method works as follows: In a first step, from time series information one has to identify critical starting areas which may lead to a crash or an outcome with low performance. In a second step, one has to perturb the system when it enters these critical areas. Due to sensitive dependence on initial conditions, already minor interventions suffice to prevent harmful events and to obtain better results.

1. Introduction

In a variety of fields, including physics, chemistry, biology, and engineering it has been demonstrated that deterministic, but nonlinear models can exhibit complicated, bounded and erratic-looking fluctuations, which are hard to distinguish from the behavior of stochastic time series. This finding, that very simple deterministic laws of motion might be responsible for the complexity around us fascinated academics and the popular press alike and became a research topic of increasing interest over the last 25 years. Economists and other researchers who study firm behavior and market dynamics got interested in this topic as well and in many contributions it has been shown that their models may also exhibit a wide range of dynamics.

Although it has been observed that sustained erratic oscillations might be beneficial e.g. for the human heart or for the human perception of temperature, for firms and economic systems more often than not it has been argued that such fluctuations are harmful or disadvantageous and, therefore, should be avoided or, if possible, even actively removed. Researchers in physics developed a number of approaches which can serve the purpose of controlling such chaotic oscillations. Some of these methods demonstrated that chaos could be controlled as soon as the system state is close to a steady state; others showed that even if the state is far away from any steady state (or cycle), it could be steered to a neighborhood by only small perturbations (e.g. [19–22]). These developments have subsequently been applied to

* Corresponding author. Tel.: +43 1 58801 33062; fax: +43 1 58801 33096.
E-mail address: kopel@imw.tuwien.ac.at (M. Kopel).

0960-0779/$ - see front matter © 2007 Published by Elsevier Ltd.
doi:10.1016/j.chaos.2007.01.015

economic models and it has been demonstrated that these techniques work quite effectively (see for example, [25,18,1,12,13,2,7]).

There might be several reasons, however, why an application of such sophisticated techniques either might not improve the system's performance or might not be even applicable, in particular for applications in economics. Let us consider the first point that taming chaotic oscillations might not be preferable to a situation without intervention. It has been shown, for example, that in economic models chaotic time paths can actually be the solutions to dynamic optimization problems and therefore be superior to steady state behavior with respect to some long-run performance measures (see e.g. [3,11]). The same observation has been made for models with (only) boundedly rational decision makers, who are interested in solving optimization problems with a rather short time horizon (this might be due to informational constraints or their information-processing capabilities); see e.g. [10,14,15]. Certainly, in such situations getting rid of fluctuations might not be a preferable choice.

In this paper, we are more interested in the second point, namely that these techniques might not even be applicable or suitable for the management of a firm or the policy maker in an economic system. It is a fact that the laws governing the dynamics of economic or market systems are not known. Furthermore, and maybe in contrast to the natural sciences, experiments are hard to conduct and the historical time series obtained from real-world observations are too short to reliably reconstruct these laws from the data. Finally, large interventions might be prohibitively costly or even impossible for managers and policy makers. Hence, even if erratic fluctuations are not desirable, these constraints oftentimes renders sophisticated controlling-techniques ineffective. What is needed in such a situation is a rather robust and simple method, which effectively improves the performance of the firm or the economic system despite those limitations.

In this paper, we introduce such an approach and illustrate its workings by applying it to an economic model introduced by Kopel [13]. Our idea is based on the following observation. If we consider the state variables of a dynamical system and a performance measure (e.g. profit, sales revenue, unemployment, the growth rate, etc.) which depends on these state variables, then certainly there will be regions in the phase space where the performance is low while in others it will be high. It might also be the case that some regions should be completely avoided because it has bad or even disastrous consequences if the trajectory passes through these regions. For example, if the output of a firm is very low then profits may be low or even negative due to fixed costs. If the output is very high, then profits may also be low due to low prices. So very low or very high production levels should be prevented by the management of a firm. We show that, without knowing the law of motion, the decision makers of a firm can discover such critical regions from time series and then make very small and infrequent interventions such that the trajectory does not pass through them. It is illustrated that this may improve the long-run performance of a firm. To have a particular example at hand, we focus on the behavior of firms. However, it should be clear that the method is more general and may be applied to a much wider range of dynamic economic systems.

2. A simple model of a firm's routines

A firm's decisions are often guided by simple routines. These well-practiced patterns of activity inside the firm include e.g. hiring procedures, policies for determining how much to produce, and how much should be advertised under various circumstances. Since decision-making is costly and decision makers are boundedly rational, these routines are often incorporated into simple "rules-of-thumb" (see e.g. [6,5]). As an illustration of how such simple decision rules might emerge in an economic framework, let us consider the following simple dynamic model.[1] Firms offer their products in a competitive market and prices are determined by market clearing (to keep the model as simple as possible all firms are assumed to be equal). At the beginning of period $t+1$ a firm's management has to decide how much to produce without knowing the price of that period. All it has is a price expectation p^e_{t+1}, which is assumed to depend on the price of the previous period. The produced output will be shipped to the market and is then sold for the actual market price. During the process of deciding how much to produce the decision makers have to consider two constraints. First, financial resources (working capital) in period $t+1$ are given by F_{t+1}. The achieved profit P_t in period t can be used either for internal financing or else can be paid out in the form of dividends. If we denote the retention rate with r, then $F_{t+1} = rP_t$. Second, firms are assumed to have access to the capital market where they can acquire short-term capital in period t which has to be paid back in the subsequent period, where the interest rate is i. Let the parameter δ denote the share of the production costs which is financed by the working capital F_{t+1} and the parameter γ denote the costs per unit of production. Suppose, furthermore, that market prices are determined by a linear inverse demand curve of the form $p_t = \alpha - \beta(nx_t)$, where nx_t denotes the total supply of the n firms in the market.

[1] For a more general version of this model the reader is referred to Kopel [13], where references to the literature and more details on the economic foundation of the assumptions are given.

Please cite this article in press as: Kopel M et al., Regulating complex dynamics in firms and economics systems, Chaos, Solitons & Fractals (2007), doi:10.1016/j.chaos.2007.01.015

Then, taking all that into account, the constrained maximization problem of each sales-revenue-maximizing firm is

$$\max_{x} \quad p^{e}_{t+1}x$$
$$\text{subject to } \delta\gamma x \leqslant rP_t,$$

where $P_t = P(x_t) = (\alpha - \beta n x_t)x_t - (1-\delta)(1+i)\gamma x_t$. The first term is the sales revenue and the second term is the share of the production costs of the previous period, which has been financed by external funds and has to be paid back in the current period. If we e.g. assume naive expectations, i.e. $p^{e}_{t+1} = p_t$, then the sales-revenue-maximizing production choice is $x_{t+1} = (r/\delta\gamma)[(\alpha - \beta n x_t)x_t - (1-\delta)(1+i)\gamma x_t]$. This difference equation incorporates in a simple way the quite complex interactions of production and financial decisions of the firm's managers. However, these managers are only boundedly rational. Their behavior is only locally, but not globally optimal since they do not take into account the global structure of environmental feedback.

In the remainder of the paper we will not refer to the details of the model. Instead, we will consider the decision rule as an example of an organizational routine, which is used to determine output. If we set $\kappa := r/\delta\gamma$, $a := \alpha$, $b := \beta n$ and $c := (1-\delta)(1+i)\gamma$, the rule reads

$$x_{t+1} = f(x_t, \kappa) = \kappa[(a - bx_t)x_t - cx_t], \tag{1}$$

where $P(x_t) = (a - bx_t)x_t - cx_t$ denotes the profit. Hence, what this routine says is that the management's future output choice is based on current profit. We will assume that the management is not willing to change the routine. This is motivated by the fact that firms do not change their routines often because getting members of an organization to alter what has been worked out in the past is difficult.

3. A simple intervention method

Fig. 1 illustrates the dynamics of our model in the time domain for $a = 20$, $b = 5$, $c = 4$ and $\kappa = 0.25$. As can be seen in the top panel, production levels vary strongly and repeatedly drop to very low levels. For instance, around period 100, the trajectory of output quantities nearly equals zero and it takes some iterations before the system reaches again higher production values. Similar drops in production can be observed repeatedly. The corresponding profits would obviously also show the same kind of erratic behavior, since the production level in period t is proportional to the profit in the previous period by the firm's decision rule. The bottom panel of Fig. 1 shows the corresponding sales revenues and, obviously, they fluctuate strongly too. Note that for some periods sales revenues are very low or even close to zero. Using 10,000 observations, the average sales revenue of the firm is 12.73.

The drops in sales revenues presented above are of course not desirable from the management's point of view. Obviously, if sales revenues fall below a certain threshold, the firm may run into liquidity problems or even go bankrupt

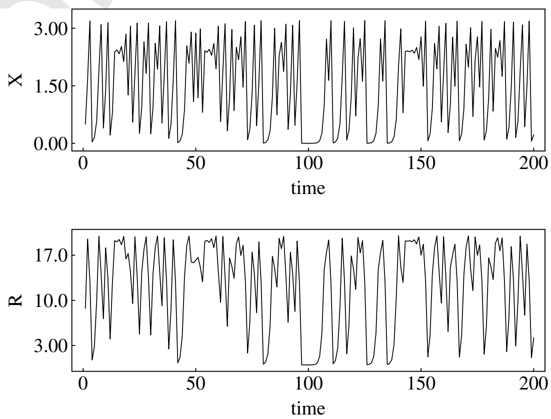

Fig. 1. The dynamics of our model with $a = 20$, $b = 5$, $c = 4$ and $\kappa = 0.25$ in the time domain for 200 observations. The top (bottom) panel displays the evolution of output (sales revenues).

Please cite this article in press as: Kopel M et al., Regulating complex dynamics in firms and economics systems, Chaos, Solitons & Fractals (2007), doi:10.1016/j.chaos.2007.01.015

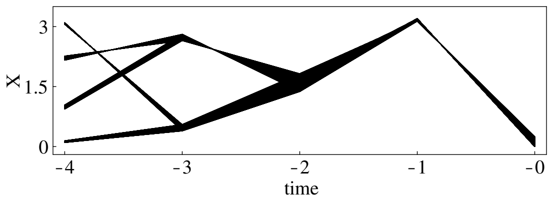

Fig. 2. Crash paths of our model between $t = -4$ and $t = 0$. A crash is defined as a drop below $x_c = 0.25$. We display 500 collected crash orbits. Parameter setting as in Fig. 1.

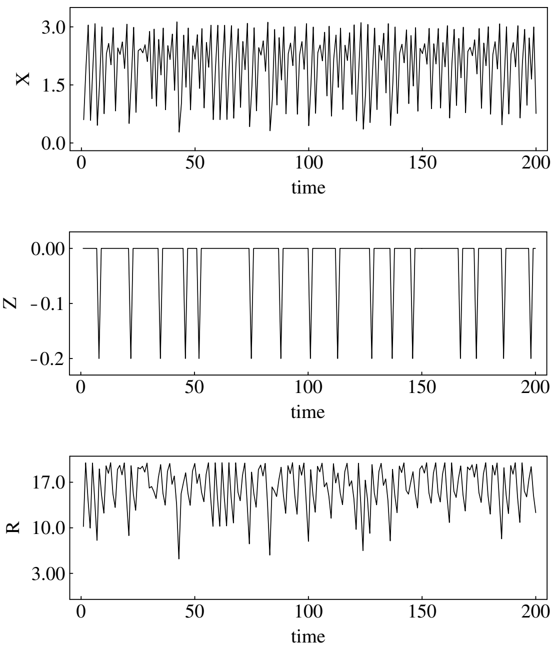

Fig. 3. The top panel displays the dynamics of the modified model with $Z = -0.2$ in the time domain (200 observations, parameter setting as in Fig. 1), the central panel shows the corresponding interventions, and the bottom panels depicts the sales revenues.

(even more so if the situation is worsened by some additional external shock in the economy). In addition, if production is very low, the firm has to lay-off most of its work force. Let us therefore assume, that the management of the firm seeks to prevent trajectories to fall below a critical value x_c, say $x_c = 0.25$.[2] It is easy to numerically compute that the probability that the output level is below this threshold level is about 18.14 percent.

Our method starts with the management collecting information on the paths of trajectories that may lead to such a crash. Then, by exploiting the sensitive dependence of initial conditions of a chaotic system, it tries to make small interventions at these critical points to avoid bigger crashes in future periods. It will turn out that all this can be achieved without knowing the actual law of motion. The management can base its intervention strategy on simple time series information. Furthermore, only rare and small interventions are needed, which is another important factor, as interventions are costly.[3]

[2] As a consequence, the management has to adapt the orgnaizational routine with respect to our proposed method. In a more general market model, policy makers intervene in order to minimize market crashes.

[3] When we talk about "costs of intervention", we (also) refer to the costs which arise by the effort the management has to devote to collect information and to monitor the intervention process and make sure that employees implement their decisions correctly. Costs also arise because changes have to be communicated and information transmission channels and procedures have to be established. Moreover, each such intervention "breaks" the corporate routine activity and puts the corporation into an "alert state". It seems plausible that a strategy which only requires few of such interventions is less expensive than a method which only works if one changes the output decision resulting from routine-driven behavior in each and every time step.

Please cite this article in press as: Kopel M et al., Regulating complex dynamics in firms and economics systems, Chaos, Solitons & Fractals (2007), doi:10.1016/j.chaos.2007.01.015

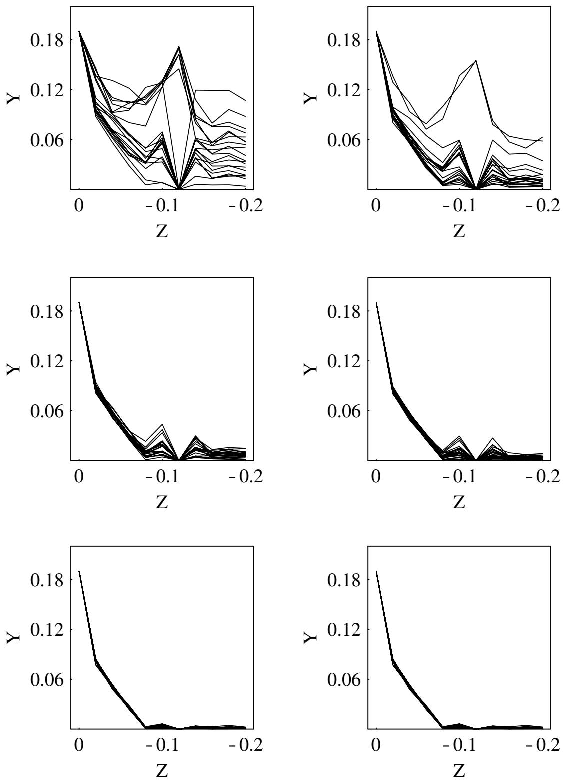

Fig. 4. The effectiveness of the method for increasing intervention size Z. We display the probability that the trajectory is below $x_c = 0.25$. In the panels from the top left to the bottom right we use 100, 200, 400, 800, 1600, and 3200 observations, respectively, to identify the alert zone for period $t - 3$. Results are presented for 20 simulation runs (10,000 observations each), generated by using different initial conditions.

Finally, it should be mentioned that it is likely that the system will remain chaotic.[4] With regard to this point one might argue that there is strong evidence that economic agents are usually risk-averse (see [8,24,16]) and that they therefore prefer a world in which chaos (and the associated strong volatility of system variables) is suppressed or removed. However, under certain circumstances chaos control may not be possible. In such situations our method may then be used to prevent at least some extreme outcomes or crashes.

Let us now turn to the first step. Collecting information in our model is straightforward. The management simply has to record production data $x_{t-1}, x_{t-2}, \ldots$ whenever $x_t < x_c$ and search for a pattern in these data. Fig. 2 illustrates the results for our map with $x_c = 0.25$. Going backwards in time, it is quite obvious that $f(x_{t-1} > 3.1367) < x_c$ and $f(1.3750 < x_{t-2} < 1.8249) > 3.1367$ and finally that $f(0.3918 < x_{t-3} < 0.5511)$ and $f(2.6493 < x_{t-3} < 2.8083)$ are $\in [1.3750, 1.8249]$, respectively. Thus, if the management observes that the output trajectory falls into the alert zone A, that is, if $x_{t-3} \in [0.3918, 0.5511]$ or $[2.6493, 2.8083]$, then it can intervene in the system in $t-2$ such that $0.25 < x_{t-1} < 3.1367$, which prevents the production level to fall below x_c in period t.[5]

To give an example, the interventions may be executed by an additional feedback term that leads to

[4] Our method is related to the one proposed by Yang et al. [26]. Their goal, however, is to maintain transient chaos, i.e. to prevent periodic orbits. In a biological context, the emergence of periodicity may be regarded as pathologic (and is referred to as a dynamical disease). In addition, their approach is not based on time series information. For some biological applications and refinements of this approach see, [4,23,17] or [9]. Overall, this literature is known under the heading "chaos anti-control".

[5] It is important to note that for the sake of demonstration we have selected one particular example where the firm intervenes *two* time steps ahead of the crash date. The firm may also decide to intervene e.g. one time step later. However, the management has to consider a trade-off: the earlier it intervenes, the smaller is the necessary adjustment of output and vice versa. So the decision makers may decide whether they want to intervene early and change output only slightly or wait and intervene more forcefully later.

Please cite this article in press as: Kopel M et al., Regulating complex dynamics in firms and economics systems, Chaos, Solitons & Fractals (2007), doi:10.1016/j.chaos.2007.01.015

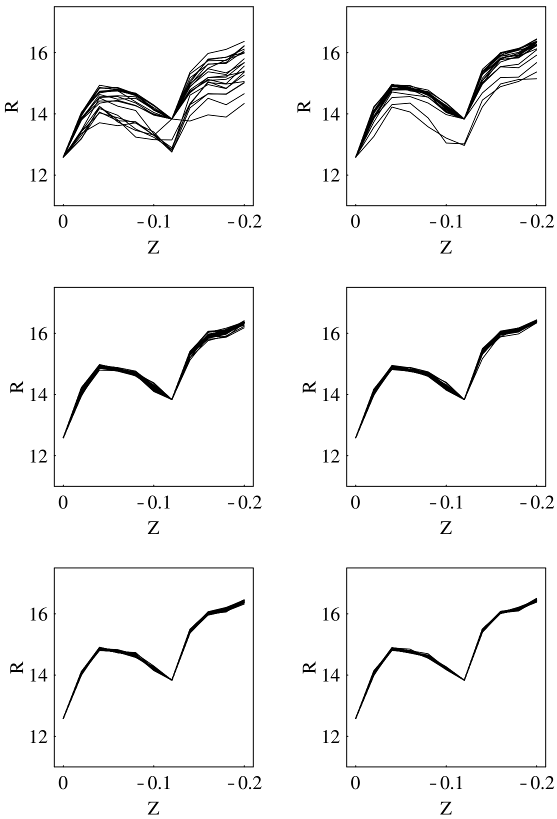

Fig. 5. The evolution of average long-term sales revenues for increasing intervention size Z. The numbers correspond to the simulation outcome presented in Fig. 4.

$$x_{t-1} = \begin{cases} f(x_{t-2}) + Z & \text{for } x_{t-3} \in [0.3918, 0.5511] \quad \text{or} \quad x_{t-3} \in [2.6493, 2.8083], \\ f(x_{t-2}) & \text{otherwise.} \end{cases} \tag{2}$$

Z is an arbitrary intervention function, which has to ensure that the orbit leaves the crash path. In Fig. 3 we simply set $Z = -0.2$. In this case the financial resources, which have been available for production, are not used up completely, and we assume that in those periods the excess amount is paid out as an additional dividend. The top panel shows the trajectory of the modified model and the panel in the middle indicates the magnitude and frequency of interventions.[6] At least in the first 200 periods, the production level does not drop below $x_c = 0.25$. Moreover, the management must intervene in just 17 cases. Infrequent interventions thus may suffice to prevent the system from dropping below the threshold value. Observe that the management does not have to change the firm's routine, but instead issues temporary and short-term modification plans. Exploring the first 10,000 observations for this specific parameter setting shows that the variable is only in less than 0.05% of the cases below x_c. Recall that without interventions, this happens in about 18.29% of the cases. Finally, the bottom panel in Fig. 3 depicts the corresponding sales revenues. Obviously, regions with extremely low sales revenues can be avoided. To the delight of the firm's management, the average sales revenues increase from 12.73 to 16.45, a positive change of about 30%. In addition, the average profits increase also by about 30% from 6.37 to 8.25.

Next, we explore the effectiveness of our intervention method with respect to an increasing number of observations available to identify the alert zone A. Since both, having frequent modifications and collecting more data, result in higher costs, it is of considerable practical importance to know how the method performs if few data points are avail-

[6] It is easy to check that the method also works for other one-dimensional nonlinear maps, i.e. the logistic map, the tent map, or the Gaussian map. The method may also be applied in higher dimensional systems, although it may become more difficult to learn the critical intervention space. For some hints on this aspect see, e.g., [26] and the references in footnote 3.

Please cite this article in press as: Kopel M et al., Regulating complex dynamics in firms and economics systems, Chaos, Solitons & Fractals (2007), doi:10.1016/j.chaos.2007.01.015

able or intervention levels are restricted to be small. In Fig. 4, we plot the probability that the system falls below the critical value x_c, denoted as Y, against the intervention size Z. In the panels from the top left to the bottom right we use 100, 200, 400, 800, 1600 and 3200 observations, respectively, to calculate the alert zone A. The results are presented for 20 simulation runs with 10,000 observations each, generated by using different randomly chosen initial conditions. The results are striking. For already 100 observations (and sufficiently large Z) it is possible for the management to significantly reduce the probability that the system drops below the critical value. Moreover, the results improve with increasing number of observations. Consequently, if the firm's management invests in a better enterprise information system, then the performance improvement is even higher. For instance, almost all crashes are avoided for a learning period of 3200 observations and a moderate intervention force.

Fig. 5 depicts how the average long-term sales revenue depends on Z. The panels directly correspond to the one displayed in Fig. 4. Given our parameter setting, $x_c = 0.25$ implies $R_c = 5$, i.e. if output does not fall below 0.25, then in

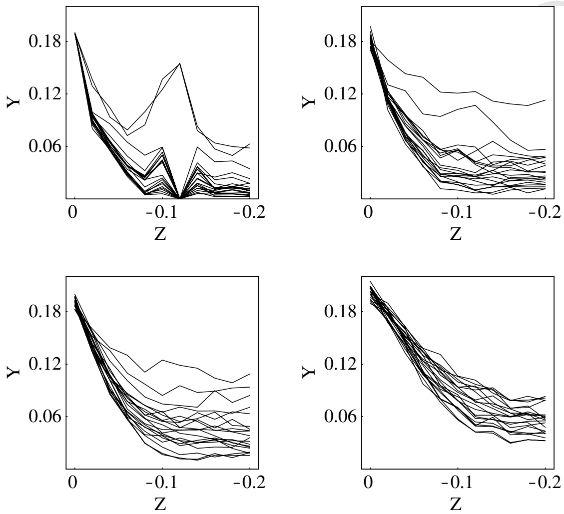

Fig. 6. The effectiveness of the method for increasing intervention size Z and different noise intensities. We display the probability that the trajectory is below $x_c = 0.25$. In the panels from the top left to the bottom right we set $\sigma = 0$, $\sigma = 0.05$, $\sigma = 0.1$, and $\sigma = 0.2$, respectively. We use 200 observations to identify the alert zone for period $t - 3$. Results are presented for 20 simulation runs (10,000 observations each), generated by using different initial conditions.

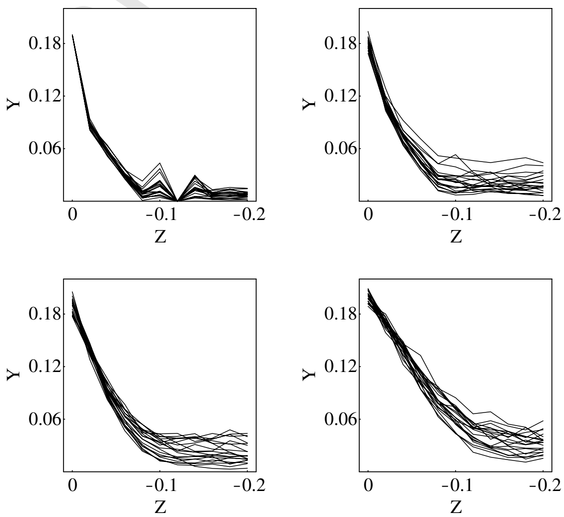

Fig. 7. The figure is based on the same simulation design as Fig. 6, except that we now use 400 observations to identify the alert zone for period $t - 3$.

Please cite this article in press as: Kopel M et al., Regulating complex dynamics in firms and economics systems, Chaos, Solitons & Fractals (2007), doi:10.1016/j.chaos.2007.01.015

each period, sales revenue will be above 5. The panels of Fig. 5 reveal that our method may furthermore increase sales revenues by about one-third. Again, already few observations seem to be sufficient to support the main goal of the firm. It is important to realize, however, that the relation between intervention strength and sales revenues is nonlinear. Even for tiny interventions, say $Z = -0.05$, sales revenues may significantly be raised.

Finally, we turn to the important issue whether the effectiveness of the proposed method is robust with respect to noise. Therefore, we add dynamic noise ε_t to the system, where ε is IID normally distributed with mean zero and constant standard deviation σ. In addition, we restrict the dynamics of the model to the interval $0 < x_t < 3.2$. The lower bound is obvious since output cannot be negative. The upper bound, which is the extreme production value one may observe in the deterministic scenario, may be regarded as maximal production capacity. The four panels of Fig. 6 are computed with $\sigma = 0$, $\sigma = 0.05$, $\sigma = 0.1$ and $\sigma = 0.2$, respectively. The panels depict the probability that the output level is below the threshold level $x_c = 0.25$ for increasing Z (as in Fig. 4). In all four cases, we have used 200 observations to identify the alert zone. Fig. 7 shows the same, except that the learning period now comprises 400 time steps. Visual inspection reveals that even in the presence of substantial exogenous noise, the probability of unwanted crashes may significantly be reduced. The longer the learning period, the better the method works. However, even for as few as 200 observations and $\sigma = 0.2$, crashes are reduced by a factor of at least 1/2 if Z is sufficiently strong. In about half of the 20 simulation runs, the probability Y even decreases by about 2/3.

4. Conclusions

Nonlinear dynamical systems may display erratic fluctuations, including regular dramatic changes. In economics, such events may stand for firms going bankrupt, severe business cycles, and stock market crashes. In this paper, we discuss a method to regulate potentially harmful chaotic dynamics. The main point of the method is to learn certain intervention points in advance of the crashes. Application of the method needs no knowledge of the underlying system dynamics but only time series information. Another advantage is that our method only requires rare and small interventions. We have demonstrated the working of the method by relying on a simple model of firm behavior. Besides preventing output to fall below a pre-specified value, it has turned out that the interventions may also increase average long-term sales revenues and average long-term profits. Our simulations also give strong evidence that the method is robust with respect to noise.

Acknowledgement

We are grateful for the critical comments of two anonymous referees, which led to a considerable improvement of our paper.

References

[1] Agiza HN. On the analysis of stability, bifurcation, chaos and chaos control of Kopel map. Chaos, Solitons & Fractals 1999;10:1909–16.
[2] Bala V, Majumdar M, Mitra T. A note on controlling a chaotic tatonnement. J Econ Behav Organ 1998;33:411–20.
[3] Dawid H, Kopel M, Feichtinger G. Complex solutions of nonconcave dynamic optimization models. Econ Theory 1997;9:427–39.
[4] Dhamala M, Lai Y-C. Controlling transient chaos in deterministic flows with applications to electrical power systems and ecology. Phys Rev E 1999;59:1646–55.
[5] Dosi G, Nelson RR. An introduction to evolutionary theories in economics. J Evol Econ 1994;4:153–72.
[6] Futia C. The complexity of economic decision rules. J Math Econ 1977;4:289–99.
[7] Gordon T, Greenspan D. The management of chaotic systems. Technol Forecast Social Change 1994;47:49–62.
[8] Géczy C, Minton BA, Schrand C. Why firms use currency derivatives. J Finance 1997;52:1323–54.
[9] Hilker F, Westerhoff F. Preventing extinction and mass outbreaks in irregularly fluctuating populations. 2006. http://arxiv.org/abs/q-bio/0606042.
[10] Huang W. Caution implies profit. J Econ Behav Organ 1995;27:257–77.
[11] Kopel M, Dawid H, Feichtinger G. Periodic and chaotic programs of intertemporal optimization models with non-concave net benefit function. J Econ Behav Organ 1998;33:435–47.
[12] Kopel M. A new view of organizations as disequilibrium systems, mimeo, Vienna University of Technology. 2001.
[13] Kopel M. Improving the performance of an economic system: controlling chaos. J Evol Econ 1997;7:269–89.
[14] Matsumoto A. Preferable disequilibrium in a nonlinear cobweb economy. Ann Oper Res 1999;89:101–23.

[15] Matsumoto A, Nonaka Y. Statistical dynamics in chaotic Cournot model with complementary goods. J Econ Behav Organ, forthcoming.
[16] Nance DR, Smith Jr CW, Smithson CW. On the determinants of corporate hedging. J Finance 1993;48:267–84.
[17] Piccardi C. On the control of chaotic systems via symbolic time series analysis. Chaos 2004;14:1026–34.
[18] Richter H, Stolk A. Control of the triple chaotic attractor in a Cournot tripoly model. Chaos, Solitons & Fractals 2004;20:409–13.
[19] Shinbrot T, Ott E, Grebogi C, Yorke JA. Using chaos to direct trajectories to targets. Phys Rev Lett 1990;65:3215–8.
[20] Shinbrot T, Ott E, Grebogi C, Yorke JA. Using chaos to direct orbits to targets in systems described by a one-dimensional map. Phys Rev A 1992;45:4165–8.
[21] Shinbrot T, Ott E, Grebogi C, Yorke JA. Using the sensitive dependence of chaos (the ‘‘butterfly effect) to direct trajectories in an experimental chaotic system. Phys Rev Lett 1992;68:2863–6.
[22] Shinbrot T, Ott E, Grebogi C, Yorke JA. Using small perturbations to control chaos. Nature 1993;363:411–7.
[23] Shulenburger L, Lai Y-C, Yalcinkaya T, Holt R. Controlling transient chaos to prevent species extinction. Phys Lett A 1999;260:156–61.
[24] Tufano P. Who manages risk? An empirical examination of risk management practices in the gold mining industry. J Finance 1996;51:1097–137.
[25] Wieland C, Westerhoff F. Exchange rate dynamics, central bank interventions and chaos control methods. J Econ Behav Organ 2005;58:117–32.
[26] Yang W, Ding M, Mandell A, Ott E. Preserving chaos: control strategies to preserve complex dynamics with potential relevance to biological disorders. Phys Rev E 1995;51:102–10.

Please cite this article in press as: Kopel M et al., Regulating complex dynamics in firms and economics systems, Chaos, Solitons & Fractals (2007), doi:10.1016/j.chaos.2007.01.015